Theory problem set: Optimal taxation, nudges, and tax salience

Behavioral Public Economics Mini-Course

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1 Subsidizing under-consumed goods

In this question, we consider consumers who undervalue some product, x, like healthy food or energy-efficient upgrades, because of incorrect beliefs or inattention. Consumers start out with some income y, and they maximize the utility function

$$\hat{U} = \hat{\alpha}\ln(x) + (y - (p - s)x - T)$$

where p is the price set by producers, s is the subsidy set by the government, T is the lump-sum tax, and $y - (p - s)x - T$ is spending on the "numeraire" good. Consumers' true utility is instead given by

$$U = \alpha\ln(x) + (y - (p - s)x - T)$$

where $\alpha \geq \hat{\alpha}$.

The choice of x can be any non-negative real number. The lump-sum tax T must balance the budget: if consumers choose $x^*(s)$ given a subsidy s, then $T(s) = s \cdot x^*(s)$. Assume that y is large enough such that the choice of x is always interior (i.e., it is characterized by a first-order condition).

1.1 For parts 1.1- 1.5, assume that the producer price is fixed because the good is produced by a price-taking firm with constant marginal costs of production (graphically, this means that the supply curve is flat). Let $x^*(s)$ be choice as a function of the subsidy. Show that

$$\frac{d}{ds}\left[\alpha\ln(x) - (p - s)x^*(s)\right] = \frac{\alpha}{p - s}.$$

1.2 Let $x^*(s)$ be choice as a function of the subsidy. Show that

$$\frac{d}{ds}\left[sx^*(s)\right] = \frac{\hat{\alpha}p}{(p - s)^2}.$$

1.3 Using the derivations above, show that the optimal subsidy must satisfy

$$\frac{p - s}{p} = \frac{\hat{\alpha}}{\alpha}.$$

What is the intuition behind this equation? Explain why this subsidy obtains the "first best"—i.e., welfare that would be achieved if no consumers misoptimized.

1.4 Suppose now that a fraction q of consumers have $\hat{\alpha} = \hat{\alpha}_L < \alpha$ and a fraction $1-q$ of consumers have a $\hat{\alpha} = \hat{\alpha}_H = \alpha$. Using 2.1 and 2.2 above, generalize 2.3 to show that the optimal subsidy must satisfy

$$\frac{p-s}{p} = q\frac{\hat{\alpha}_L}{\alpha} + (1-q).$$

Explain the intuition, and explain why this subsidy can no longer achieve first-best welfare. (The government must still maintain a balanced budget, meaning that the lump-sum tax T must equal the total amount of subsidy paid out to the two types of consumers. That is, $T = s\left[qx_L^* + (1-q)x_H^*\right]$,where x_L^* and x_H^* are the respective consumption choices of type L and H consumers.)

1.5 Suppose that "poor" consumers have $\hat{\alpha} = \hat{\alpha}_L$ while "rich" consumers have $\hat{\alpha} = \hat{\alpha}_H$. This means that rich people buy more x than poor people. A progressive politician named Bennie Xanders gives a speech about how the subsidy calculated in 2.4 above would be regressive, since it would end up being distributed to the rich much more than to the poor. Using your calculations in 2.4, explain why Bennie's concept of regressivity is incorrect, and that in fact the subsidy benefits the rich and the poor consumers equally. Explain the intuition behind why standard concepts of regressivity have to be refined with behavioral consumers.

1.6 For the rest of this question (1.6-1.10), assume that the supply of good x is fixed. The firm supplying this good is still a price-taking firm. (Graphically, this means that the supply curve is a vertical line.) Explain why when $q = 1$ (all consumers are homogeneous), the first-best welfare obtains with no subsidy. Unless you prefer, you don't have to do any math here. Hint: You may find it helpful to recall our discussion about how tax salience does not affect deadweight loss of taxation with vertical supply, since it does not affect equilibrium quantity sold and purchased.

1.7 Explain why, when $0 < q < 1$, the first-best welfare no longer obtains. But also explain why subsidizing (or taxing) can't improve things. Hint: Think about the principles of misallocation due to heterogeneous consumer bias that we discussed in the context of sales tax salience. Unless you prefer, you don't have to do any math here.

1.8 Go back to the case in which $q = 1$. Imagine a "nudge" that fully debiases consumers. Explain why this nudge would increase profits of the price-taking firm and decrease consumer surplus. (Consider doing this graphically by drawing the supply and demand curves.) Would the result of the nudge be different if instead the supply curve was flat?

1.9 Go back to the case in which $q = 1$. Imagine a "nudge" that debiases half of the consumers. Explain why this nudge would lower the sum of consumer and producer surplus (i.e., would lower social welfare). Hint: Use 1.6 and 1.7. Would the result of the nudge be different if instead the supply curve was flat?

1.10 Building on 2.8 and 2.9, explain why, generally speaking, "nudges" are unlikely to be "choice-preserving" policy tools (i.e., do not affect consumers' choice-sets) in markets with endogenous prices.

2 Tax salience and welfare

Consider a set-up similar to problem 1, where consumers have *true* utility given by $U(x) = u(x) + (y - (p+t)x)$. Suppose that instead of maximizing U, consumers choose x to maximize $u(x) + (y - (p + \theta t)x)$, where θ is the tax salience parameter. Suppose that the government wants to maximize social welfare, given by $W(t) = U(x^*) + \lambda t x^*(t)$, where $tx^*(t)$ is the tax revenue and $\lambda \geq 1$ is the value of public funds. Assume that $u(x) = ax - \frac{1}{2}x^2$, for $a > p$.

2.1 For a consumer with salience parameter θ, show that the demand is given by $x^*(t) = \max(a - p - \theta t, 0)$

2.2 Prove that for $p + \theta t < a$, $\frac{d}{dt}U(x^*(t)) = -x^* + (1-\theta)\theta t = p + \theta t - a + (1-\theta)\theta t$.

2.3 Prove that the tax t that maximizes $W(t)$ is $t = \frac{(\lambda-1)(a-p)}{2\theta(\lambda-1)+\theta^2}$ (assuming $a > p + \theta t$ at that value), and thus that the optimal tax is decreasing in θ. Prove also that $\max_t W(t)$, the maximum obtainable welfare, is decreasing in θ. What is the intuition?

2.4 Suppose now that a fraction α of consumers have $\theta = \theta_l$ and a fraction $1 - \alpha$ of consumers have $\theta = \theta_h$. Let social welfare be $W = \alpha U(x_l^*) + (1-\alpha)U(x_h^*) + \lambda[\alpha t x_l^*(t) + (1-\alpha) t x_h^*(t)]$, where x_l^* and x_h^* are the demands of type θ_l and θ_h consumers, respectively. Prove that the tax t that maximizes W is $t = \frac{(\lambda-1)(a-p)}{2E[\theta](\lambda-1)+E[\theta]^2+Var[\theta]}$, where $E[\theta]$ is the average θ in the population and $Var[\theta]$ is its variance.

2.5 Together, parts c and d show that holding constant the average value of θ in the population, the optimal tax is decreasing in the variance of θ. What is the intuition?

2.6 Prove that even when $E[\theta] < 1$, welfare would be higher if both types of consumers perceived the tax correctly if θ_h is sufficiently high to make $Var[\theta] > 1$. Draw a contrast to your answer in 2.3, and discuss the intuition.